Transonic Time-Response Analysis of 3-Degree-of-Freedom Conventional and Supercritical Airfoils

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Aeroelastic time-response analyses are performed for two conventional airfoils, NACA 64A006 and NACA 64A010, and one supercritical airfoil, MBB A-3, in small-disturbance transonic flow. Response results for forces and displacements were obtained by simultaneously integrating the structural equations of motion with the unsteady aerodynamic forces computed using two transonic codes: LTRAN2-NLR and USTS. Three-degrees-of-freedom—plunge, pitch, and aileron pitch—are considered. Flutter analyses are first performed, with the results used as a guideline for time-response parameter selection. Time-response results are presented showing that for each case the flight speed used to obtain neutrally stable responses is either exactly or nearly the same as the flutter speed determined in the separate flutter analysis. Effect of response amplitudes is investigated. Applicability and limitations of the two transonic codes are evaluated, compared, and discussed. Transonic time-response behavior of these airfoils is physically interpreted and discussed, and comparisons are made.

Nomenclature

a_h, b, c, c_β	= defined in Fig. 1
c_l, c_m, c_n	= coefficients of lift, moment about pitching axis, and aileron moment about hinge axis, respectively
c_p^*	= critical pressure coefficient
h	= plunging degree-of-freedom
k_c	= reduced frequency based on full chord $\omega c/U$
$[K]$, $[M]$	= stiffness and mass matrices, respectively
m	= mass of the airfoil per unit span
M	= freestream Mach number
$\{p\}$, $\{u\}$	= aerodynamic load and displacement vectors, respectively
r_α, r_β	= radii of gyration of airfoil about elastic axis and of aileron about hinge axis, respectively
U	= freestream velocity
U^*	= nondimensional flight speed $U/b\omega_\alpha$
x_α, x_β	= defined in Fig. 1
α, β	= airfoil and aileron pitching degrees of freedom, respectively
Δc_l, Δc_m, Δc_n	= differences between c_l, c_m, and c_n and their mean values in forced motion, respectively
θ_h, θ_α, θ_β	= phase angles of plunging, pitching, and aileron pitching, respectively
μ	= mass ratio $m/\pi\rho b^2$
ξ	= nondimensional plunging degree-of-freedom h/b
ρ	= freestream air density
$\phi_{h,\alpha}$, $\phi_{h,\beta}$	= phase differences between plunging and pitching or plunging and aileron pitching, respectively
ω, ω_r	= flutter and reference frequencies, respectively
ω_h, ω_α, ω_β	= uncoupled natural frequencies of plunging, pitching about elastic axis, and aileron pitching about hinge axis, respectively

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Introduction

THERE presently exist many calculation methods for determining the transonic flowfield around two-dimensional airfoils. A state-of-the-art review of these methods was given by Ballhaus and Bridgeman.[1] Following the rapid development of computational methods for transonic aerodynamics, studies on aeroelastic applications have been advanced significantly in recent years. Transonic flutter analyses of airfoils have been extensively investigated. A detailed account of these efforts has been reported, for example, by Yang et al.[2] Zwaan[3] has discussed the transonic flutter characteristics of wings, with emphasis on the transonic dip phenomenon in the flutter boundaries.

Along with transonic flutter analyses, aeroelastic time-response studies have recently attracted increasing attention. In the time-response analysis, the assumption of linear superposition of airloads is not needed. Thus such analysis may be used to check data obtained from a separate flutter analysis where linear superposition is assumed. Also, shock wave motions are accounted for.

The study of time responses, stable, unstable, or neutrally stable, provides more complete information in addition to that given in the eigenvalue flutter analysis. This information is useful for a better understanding of transonic aeroelastic problems. Such a study is a logical first step if the effect of control forces on airfoil time responses is to be studied.

Ballhaus and Goorjian[4] first reported a time-response analysis of a NACA 64A006 airfoil oscillating with a single pitching degree of freedom (DOF) at $M=0.88$. The structural equation of motion was integrated with a simultaneous update of the airloads computed from their LTRAN2 code.

Rizzetta[5] used the LTRAN2 code to perform a time-response analysis for a NACA 64A010 airfoil oscillating with a single pitching DOF and 3DOF—plunge, pitch, and aileron pitch. No attempt was made to obtain the neutrally stable response curves that indicate the flutter condition for the 3DOF system.

Guruswamy and Yang[6] used LTRAN2 to perform a time-response analysis of a flat plate and NACA 64A006 airfoil at $M=0.7$ and 0.85, respectively, with 2DOF, plunge and pitch. It was shown that the flutter speed selected from the bottom of the flutter boundary obtained in the flutter analysis indeed resulted in neutrally stable responses.

Yang and Chen[7] used the LTRAN2-NLR code (an improved version of LTRAN2 by Houwink and van der Vooren of the National Aerospace Laboratory, the Netherlands[11]) to perform time-response analyses of a conventional NACA 64A006 airfoil and a MBB A-3 supercritical airfoil oscillating with 3DOF at $M=0.85$ and 0.765, respectively. The flight speeds used to obtain all of the neutrally stable responses were found to be close to the flutter speeds obtained in the flutter analysis.

Borland and Rizzetta[8] have recently used the XTRAN3S code for time response of three-dimensional wings in transonic flow. A flutter boundary for a rectangular wing of parabolic arc cross section was determined by computing displacement response histories at different values of freestream dynamic pressure.

Since the response study of Ref. 7 was conducted using the LTRAN2-NLR code only, it is intended in this research to continue that study using the USTS code (unsteady transonic small perturbation code by Isogai of the National Aerospace Laboratory, Japan[12]) in addition to LTRAN2-NLR. This study is motivated by the fact that by using the two different two-dimensional transonic codes simultaneously, the results can be evaluated through comparison. This work is further motivated by the need to study additional cases to obtain more complete data for a better understanding of the 3DOF response problem. This study also serves as a necessary step toward new research on the responses of conventional and supercritical airfoils, including active controls for flutter suppression using both transonic codes.

For the NACA 64A006 airfoil, the response results using LTRAN2-NLR in Ref. 7 are again obtained here using USTS. In addition, the effect of amplitude on responses is studied here with USTS.

Since neutrally stable responses for the NACA 64A010 airfoil have so far not been obtained, it is of interest to study this airfoil in the case of $M=0.825$ using LTRAN2-NLR.

For the MBB A-3 airfoil, a response study was conducted in Ref. 7 using LTRAN2-NLR at $M=0.765$ and $\alpha=0.86$ deg (equivalent to $c_l \simeq 0.58$). In the present study, the conditions at $M=0.765$ and $\alpha=0$ deg are considered and both codes are used. Furthermore, smaller time steps have been used here to improve the response results obtained using LTRAN2-NLR in Ref. 7.

Aeroelastic Equations of Motion

The sign conventions and aeroelastic parameters for a typical airfoil section oscillating with 3DOF are illustrated in Fig. 1.

Considering the inertia, elastic, and aerodynamic forces, the equations of motion can be written as

$$[M]\{\ddot{u}\}+[K]\{u\}=\{p\} \tag{1}$$

where the dot denotes differentiation with respect to nondimensional time ωt and $\omega_r=\omega_\alpha$:

$$[M]=\begin{bmatrix} 1 & x_\alpha & x_\beta \\ x_\alpha & r_\alpha^2 & (c_\beta-a_h)x_\beta+r_\beta^2 \\ x_\beta & (c_\beta-a_h)x_\beta+r_\beta^2 & r_\beta^2 \end{bmatrix} \tag{2a}$$

$$[K]=\left(\frac{2\omega_r}{U^* k_c \omega_\alpha}\right)^2 \begin{bmatrix} (\omega_h/\omega_r)^2 & 0 & 0 \\ 0 & r_\alpha^2(\omega_\alpha/\omega_r)^2 & 0 \\ 0 & 0 & r_\beta^2(\omega_\beta/\omega_r)^2 \end{bmatrix} \tag{2b}$$

$$\{p\}=\left(\frac{4}{\pi\mu k_c^2}\right)\begin{Bmatrix} -c_l \\ 2c_m \\ 2c_n \end{Bmatrix} \qquad \{u\}=\begin{Bmatrix} \xi \\ \alpha \\ \beta \end{Bmatrix} \tag{2c}$$

These equations are valid under the assumptions that the airfoil is rigid and that the amplitude of oscillation is small.

Response Solution Procedure

A direct integration method based on a linear variation of acceleration[9] was employed to find the time-history dynamic responses of the aeroelastic system. Equations and details for the response solution procedure are given in Ref. 7.

In the present study, the steady solution is used as the starting condition and the aerodynamic equation alone is integrated in time for several cycles of forced oscillation. The airfoil displacements for forced motion are specified by the flutter mode

$$\begin{aligned} \xi(t)&=\xi_0\sin(\omega t+\theta_h) \\ \alpha(t)&=\alpha_0\sin(\omega t+\theta_\alpha) \\ \beta(t)&=\beta_0\sin(\omega t+\theta_\beta) \end{aligned} \tag{3}$$

determined from a U-g method flutter analysis. $\xi_0:\alpha_0:\beta_0$ and θ_h, θ_α, and θ_β are the amplitude ratio and the phase angles of the 3DOF which were obtained from the flutter solution. After the aerodynamic force and moments become periodic, the forced motion is stopped and the simultaneous integration is begun.

Transonic Codes

LTRAN2-NLR

The original transonic code LTRAN2 by Ballhaus and Goorjian[10] was constructed to time accurately integrate the two-dimensional, transonic, small-disturbance equation with a low-frequency approximation. The code has subsequently been used for aeroelastic applications.[4-6] The low-frequency limitation was later improved in its NLR version[11] when the time-derivative terms in the flow tangency condition, the expression for the pressure coefficient, and the wake boundary condition were added. However, the ϕ_{tt} term in the potential equation is still neglected. The LTRAN2-NLR code has recently been used for aeroelastic applications.[7]

A 79 (vertical) by 99 (horizontal) finite difference mesh was employed in the present aerodynamic computations using LTRAN2-NLR. The steady solution was obtained by the successive line over-relaxation method including multiple grid computations to accelerate the rate of convergence. The unsteady aerodynamic calculations were carried out using the time-integration method. Time-step size is based on accuracy, not on stability considerations since the alternating-direction implicit finite difference scheme of LTRAN2-NLR is unconditionally stable.

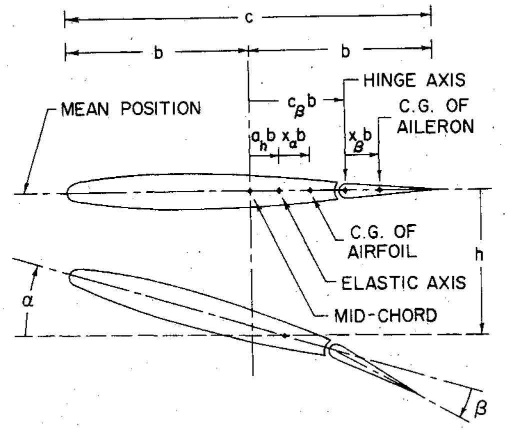

Fig. 1 Definition of parameters for 3DOF aeroelastic analysis.

USTS

The USTS code was developed by Isogai.[12] The ϕ_{tt} term is included in the potential equation of this small perturbation code which can be used for a wide range of reduced frequency ($0 \leq k_c \leq 1.0$) and for the entire transonic Mach number range (from subcritical Mach number to Mach 1). USTS was used by Isogai[12] to perform a flutter analysis for a NACA 64A010 airfoil with plunge and pitch DOF.

A 79 (vertical) by 91 (horizontal) finite difference mesh was employed in the present aerodynamic calculations using USTS. In this code, a physical coordinate plane with a smoothly stretched distribution of grid points is transformed into a computational coordinate plane with constant mesh spacing for aerodynamic computations.

Using USTS, both the steady-state and the unsteady computations were made using the time-integration method. For steady calculations, no relaxation method is available in USTS, nor are multiple grid systems, to accelerate the rate of convergence. Also, there is a time-step size restriction because the semi-implicit/implicit two-sweep finite difference procedure of USTS is conditionally stable.

Results and Discussion

Transonic time-response analyses were carried out for three airfoil configurations: 1) NACA 64A006, 2) NACA 64A010, and 3) MBB A-3. All three airfoils were among those proposed by AGARD for aeroelastic applications of transonic unsteady aerodynamics. The airfoil coordinates used were taken from Ref. 13. Aeroelastic parameter values chosen for all response cases are listed in Table 1.

Unsteady aerodynamic coefficients were first computed for a flutter analysis to determine the flutter speed U^*, flutter mode, and mass ratio μ. All three airfoils were fitted with trailing edge ailerons of 25% chord.

NACA 64A006 Airfoil

Time-response results were obtained for the NACA 64A006 airfoil at zero mean angle of attack for $M=0.7$ and 0.85 using only USTS. Amplitude effects were investigated at $M=0.85$ by increasing the amplitudes ξ_0, α_0, and β_0 by a constant factor. All response cases for this airfoil were performed at $k_c=0.3$, which required 480 time steps per cycle of motion for numerical stability.

For $M=0.7$, a 61×61 finite difference mesh was used. This high subsonic but subcritical Mach number was chosen to test the direct-integration time-response procedure in USTS. Based on the flutter speed determined by the separate flutter analysis, $U^*=2.871$ at $\mu=46.1$, the neutrally stable responses were indeed obtained. These results compare well with those of Yang and Chen,[7] who reported neutrally stable responses at $U^*=2.816$ and $\mu=48$ using LTRAN2-NLR.

For $M=0.85$, the 79×91 mesh was used. This Mach number is in the neighborhood of a transonic dip of flutter speed for the present parameters selected.

The steady pressure distributions determined by both USTS and LTRAN2-NLR are shown in Fig. 2, along with the experimental results of Tijdeman.[14] All three sets of results compare well except near 30-40% chord, and they all indicate the presence of a weak shock wave at midchord.

USTS unsteady aerodynamic coefficients were obtained for five values of reduced frequency $k_c=0.2$, 0.3, 0.4, 0.6, and 0.8 using amplitudes of oscillation $h=0.01b$, $\alpha=0.1$ deg, and $\beta=0.1$ deg. U-g method flutter analyses were then performed using these coefficients to obtain the flutter speed, amplitude ratio, and phase differences for a range of mass ratios at $M=0.85$. These results are shown in Fig. 3.

Only the bending-torsion branch of the flutter boundary appears for this set of aeroelastic parameters. The flutter speed increases with μ as in binary bending-torsion flutter. The plunge-pitch and plunge-aileron pitch amplitude ratios approach constant values (first natural mode) and the two phase differences approach zero as the mass ratio becomes larger.

From Fig. 3, a flutter condition at $k_c=0.3$ was chosen. The corresponding values of U^* and μ were 2.390 and 56.3, respectively. These values are in good agreement with Ref. 7, where a flutter solution of $U^*=2.436$ at $\mu=52.1$ was reported using LTRAN2-NLR. Flutter and time-response results for all cases in the present study are listed in Table 2. ξ_0 was set equal to 0.01 in this study.

Table 1 Aeroelastic parameter values for time-response analyses

$\omega_h/\omega_\alpha=0.3$	$\omega_\beta/\omega_\alpha=1.5$	$a_h=-0.2$	$c_\beta=0.5$
$x_\alpha=0.2$	$r_\alpha=0.5$	$x_\beta=$ 0.008	$r_\beta=0.06$

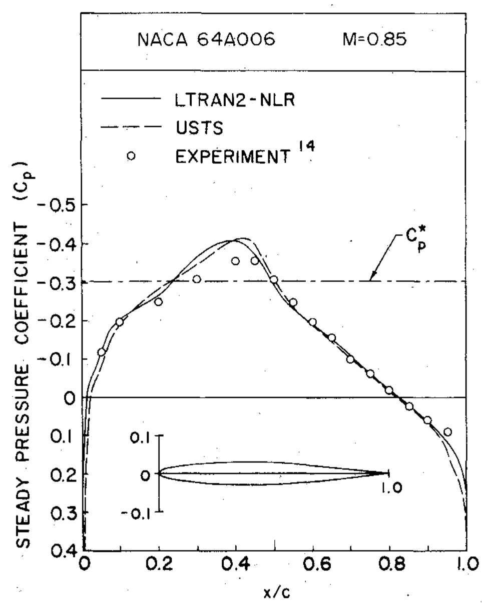

Fig. 2 Distribution of steady pressure coefficients for the NACA 64A006 airfoil at $M=0.85$.

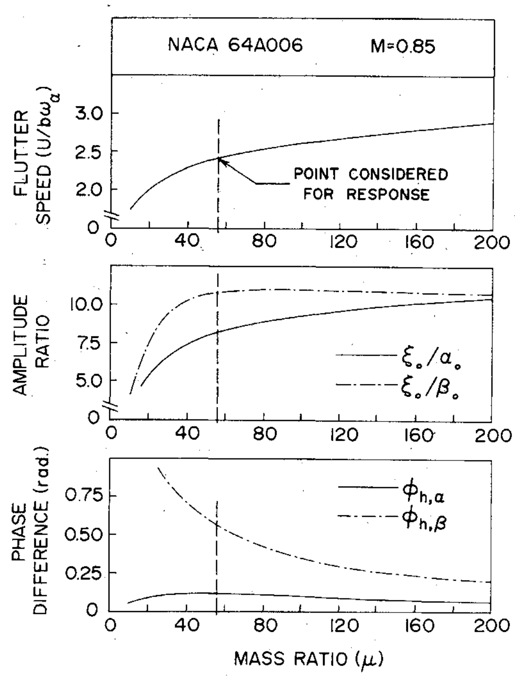

Fig. 3 Effect of mass ratio on flutter speed and flutter mode (amplitude ratio and phase angle).

Table 2 Flutter and time-response analyses results

Airfoil	Transonic code	Mass ratio μ	Flutter speed U^*	Amplitude ratio α_0/ξ_0	Amplitude ratio β_0/ξ_0	Flight speed U^* Stable	Flight speed U^* Neutrally stable	Flight speed U^* Unstable
NACA 64A006	USTS	56.3	2.390	0.122	0.0931	2.151	2.390	2.629
NACA 64A010	LTRAN2-NLR	20.86	2.464	0.748	0.638	2.415	2.452	2.488
MBB A-3, $\alpha=0$ deg	LTRAN2-NLR	51.7	2.615	0.198	0.0953	2.354	2.615	2.877
	USTS	48.9	2.588	0.186	0.0980	2.271	2.523	2.775
MBB A-3, $c_l \approx 0.58$	LTRAN2-NLR	57.5	2.459	0.145	0.106	2.213	2.459	2.705

Using parameters from the fluter analysis, a time-response analysis was carried out for the NACA 64A006 airfoil at $M=0.85$ by USTS. The airfoil was first forced to oscillate according to the prescribed amplitude ratio and phase angles of the flutter mode. After several cycles of forced motion, the three aerodynamic coefficients c_l, c_m, and c_n became periodic. Then the airfoil motion and aerodynamic responses were left free to drive each other and the aeroelastic time responses were calculated. The displacement and aerodynamic responses for the last cycle of forced motion and the first three cycles of free motion are shown in Figs. 4 and 5, respectively. In Fig. 5, c_{l_0}, c_{m_0} and c_{n_0} are the amplitudes of c_l, c_m, and c_n, respectively, obtained in the forced harmonic motion.

Based on the flutter solution, the time responses were indeed found to be neutrally stable. These results are obtained without the use of the principle of superposition of airloads which was used in the flutter analysis. Converging (stable) and diverging (unstable) responses were also obtained by using flight speeds as listed in Table 2. As compared with the neutrally stable responses, the stable and unstable response curves for aileron pitching angle β, moment coefficient c_m, and aileron moment coefficient c_n showed very small initial disturbances in the first cycle of free motion due to the initial conditions. These disturbances dissipated quickly and smooth oscillatory response behavior was found.

In addition, the frequency for stable responses is higher and the frequency for unstable responses is lower than that of the neutrally stable curves.

Using the parameters that produced neutrally stable responses for the NACA 64A006 airfoil at $M=0.85$, amplitude effects were investigated by performing a time-response analysis at multiples of the flutter mode amplitudes. Displacement and aerodynamic responses for both 10 and 30 times ξ_0, α_0, and β_0 are plotted in Figs. 6 and 7, respectively. Also depicted for comparison are the sinusoidal neutrally stable responses from Figs. 4 and 5. The "bar" denotes the increased amplitude in forced motion for each case.

Time responses computed at larger airfoil motion amplitudes illustrate the nonsinusoidal response behavior that can result from large shock wave motions. Ballhaus and Goorjian[4] and Carretta et al.[15] have presented time responses of a NACA 64A006 airfoil oscillating with a single pitching DOF that demonstrate these nonlinear unsteady effects.

In the present study, the large amplitude responses computed for the NACA 64A006 airfoil at $M=0.85$ by USTS appear to remain neutrally stable. The pitching moment coefficient in Fig. 7 is computed about the quarter chord. The amplitude of the moment coefficient is $c_{m_0}=0.0015$ and for 30 times ξ_0, α_0, and β_0, a small phase shift is observed in moment coefficient. It is noted that pitching moment responses about the elastic axis (40% chord) are very sensitive to amplitude change due to the relatively small c_m amplitude found there ($c_{m_0}=0.00043$).

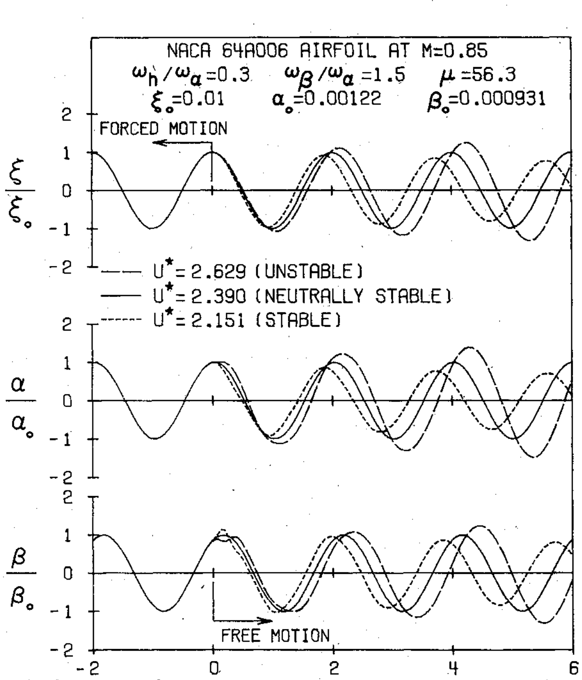

Fig. 4 Effect of flight speed on displacement responses for the NACA 64A006 airfoil at $M=0.85$ by USTS.

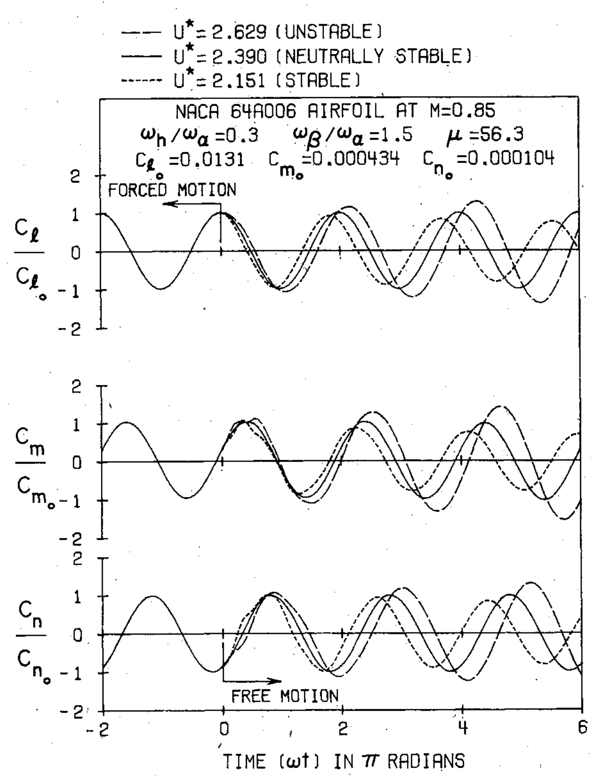

Fig. 5 Effect of flight speed on aerodynamic responses for the NACA 64A006 airfoil at $M=0.85$ by USTS.

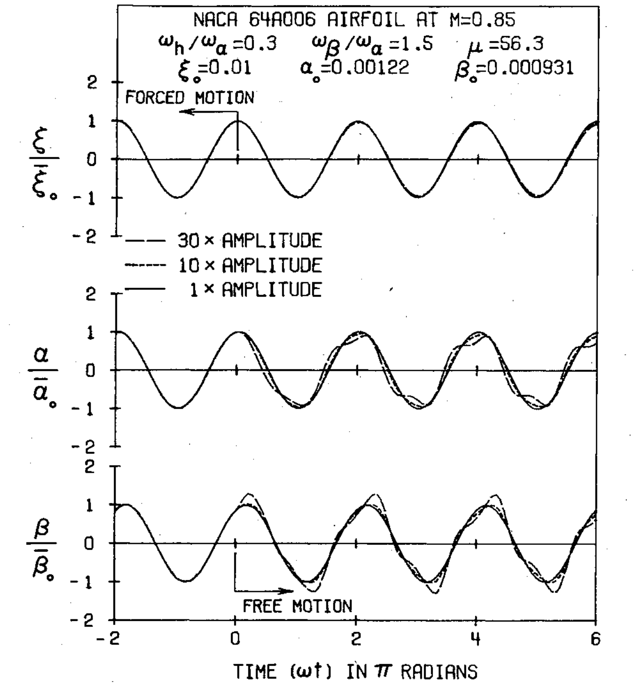

Fig. 6 Effect of amplitude on displacement responses for the NACA 64A006 airfoil at $M=0.85$ by USTS.

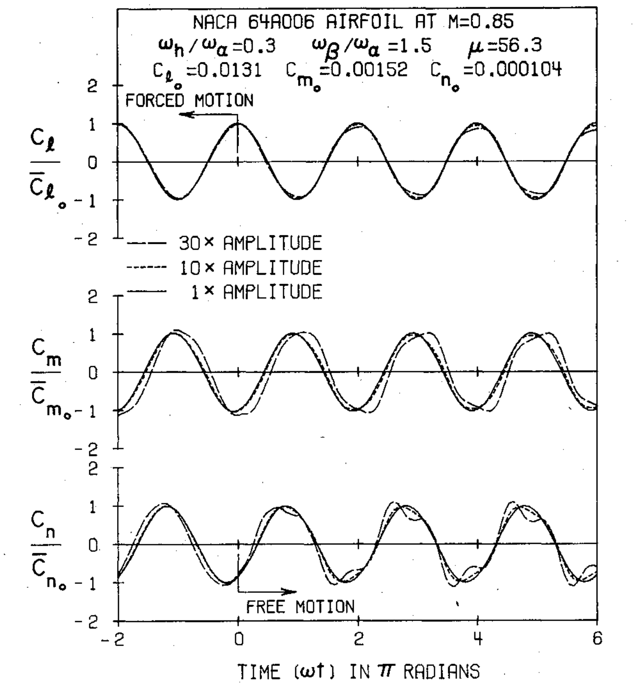

Fig. 7 Effect of amplitude on aerodynamic responses for the NACA 64A006 airfoil at $M=0.85$ by USTS.

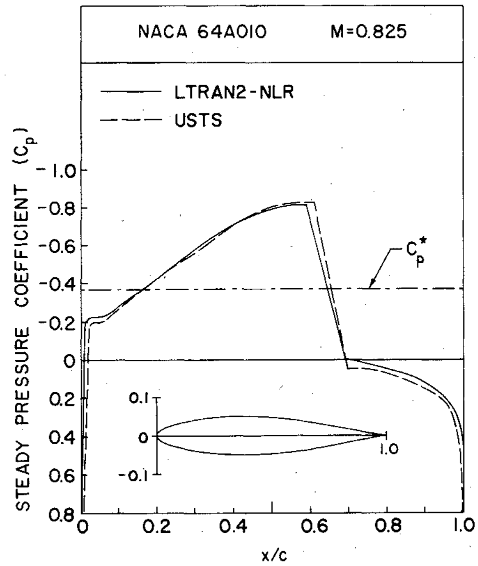

Fig. 8 Distribution of steady pressure coefficients for the NACA 64A010 airfoil at $M=0.825$.

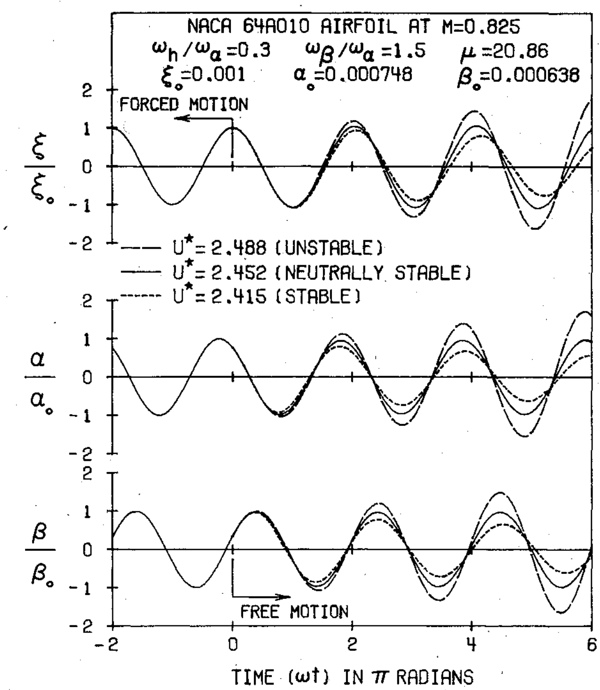

Fig. 9 Effect of flight speed on displacement responses for the NACA 64A010 airfoil at $M=0.825$ by LTRAN2-NLR.

NACA 64A010 Airfoil

Time-response analyses were carried out for the NACA 64A010 airfoil at $M=0.825$, where for the present parameters selected, the flutter speeds are in the neighborhood of a transonic dip. Isogai[12] also performed flutter analyses for this airfoil at $M=0.825$ using USTS, but with 2DOF.

The reduced frequency was selected as $k_c=0.5$. Following Rizzetta,[5] 360 time steps per cycle were used in LTRAN2-NLR for the simultaneous integration procedure, as well as for computing unsteady aerodynamic coefficients.

Figure 8 shows the steady pressure distributions computed by both LTRAN2-NLR and USTS. Results between the two codes compare very well. A relatively strong shock wave is present near 65% chord.

Time-response results were obtained by LTRAN2-NLR for the NACA 64A010 airfoil using parameters from the separate flutter analysis. All numerical results have been listed in Table 2. Displacement and aerodynamic responses for the last cycle of forced motion and the first three cycles of free motion are shown in Figs. 9 and 10, respectively. Based on the flutter solution, slightly diverging responses were obtained. Neutrally stable responses resulted when the flight speed was decreased by 0.5%. Stable and unstable time histories obtained by changing the flight speed are also shown in Figs. 9 and 10. In these responses, the frequency was found to be very close to that of the neutrally stable free motion. Also, no disturbances occurred in the first cycle of free motion when the initial conditions for simultaneous integration were imposed.

Neutrally stable USTS time responses were not obtained for the NACA 64A010 airfoil at $M=0.825$ due to numerical convergence difficulties resulting in only fairly periodic responses.

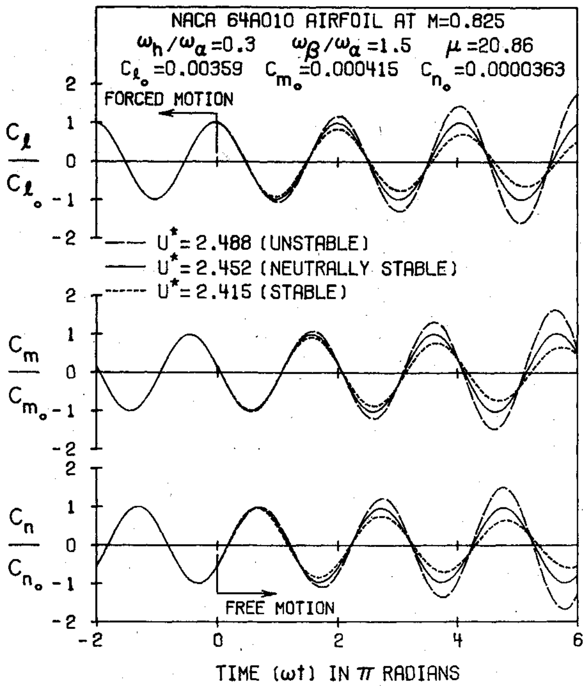

Fig. 10 Effect of flight speed on aerodynamic responses for the NACA 64A010 airfoil at $M = 0.825$ by LTRAN2-NLR.

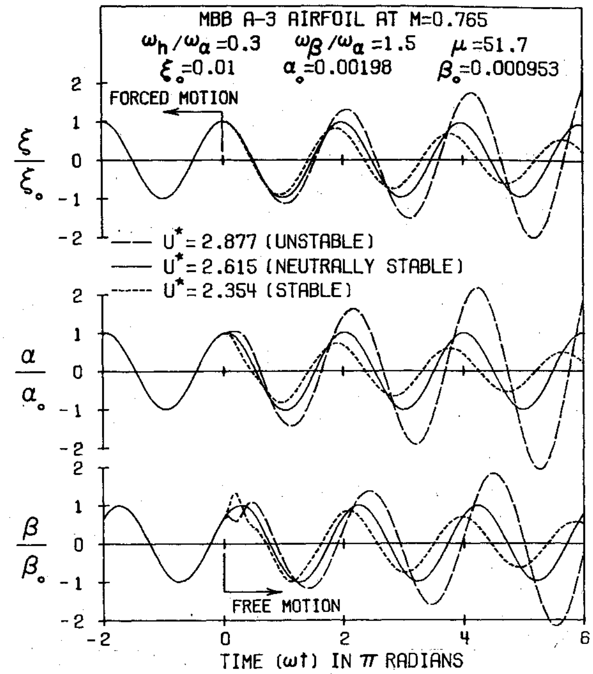

Fig. 12 Effect of flight speed on displacement responses for the MBB A-3 airfoil at $M = 0.765$ and $\alpha = 0$ deg by LTRAN2-NLR.

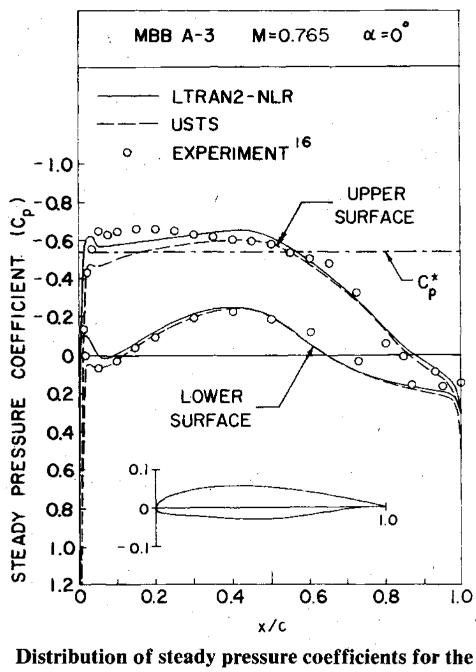

Fig. 11 Distribution of steady pressure coefficients for the MBB A-3 airfoil at $M = 0.765$ and $\alpha = 0$ deg.

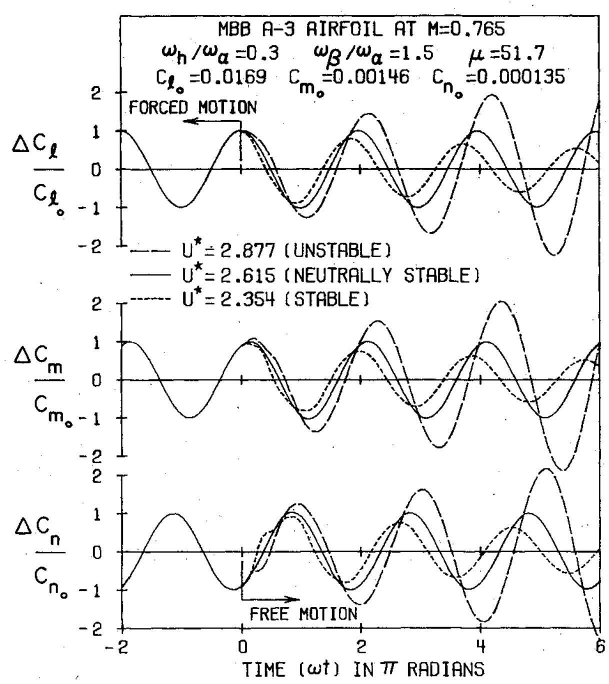

Fig. 13 Effect of flight speed on aerodynamic responses for the MBB A-3 airfoil at $M = 0.765$ and $\alpha = 0$ deg by LTRAN2-NLR.

MBB A-3 Supercritical Airfoil

Time-response results were obtained for the MBB A-3 supercritical airfoil at the design Mach number $M = 0.765$ and $k_c = 0.3$ to study the response behavior: 1) at zero mean angle of attack, and 2) at the design steady lift coefficient $c_l \approx 0.58$. Response characteristics of the MBB A-3 airfoil at the design Mach number are of special importance since the transonic dip phenomenon has been observed in the neighborhood of $M = 0.765$ for the parameters selected in Ref. 7.

For USTS, 480 time steps per cycle of motion were used to compute the unsteady aerodynamic coefficients and time-responses. For LTRAN2-NLR, 120 time steps per cycle were used to compute unsteady aerodynamic coefficients while 360 time steps per cycle were required for the simultaneous response procedure.

Case 1: At Design Mach Number of 0.765 and $\alpha = 0$ deg

Figure 11 shows the steady pressure distributions for the MBB A-3 airfoil at $M = 0.765$ and $\alpha = 0$ deg computed by both codes. The steady lift coefficients that resulted were $c_l = 0.378$ and 0.343 by LTRAN2-NLR and USTS, respectively. The experimental results of Bucciantini et al.[16] obtained at the Polytechnic of Turin Wind Tunnel for $M = 0.751$, $\alpha = 1.12$ deg, and $c_l = 0.368$ were also plotted. A weak shock wave is present on the upper surface near 55% chord.

Results agree reasonably well with the experimental data except in the neighborhood of 5-25% chord for the upper surface pressure distribution. The present zero angle of attack is, however, smaller than that used in Ref. 16, resulting in decreased upper surface pressures just aft of the leading edge

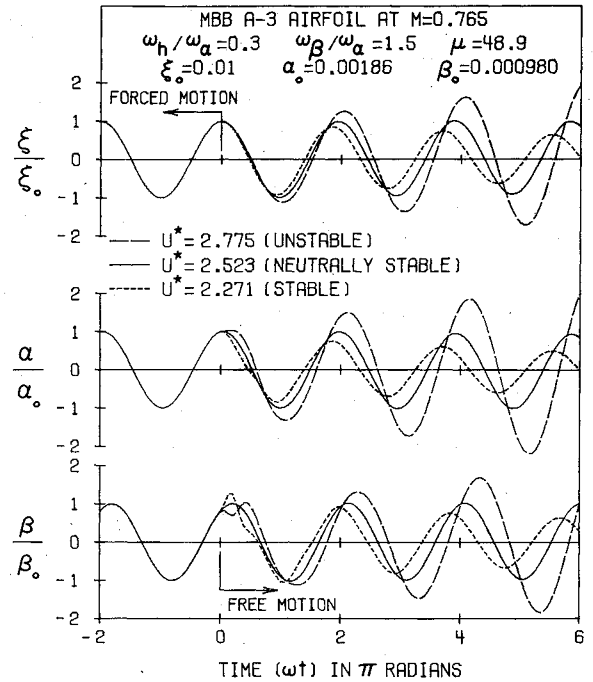

Fig. 14 Effect of flight speed on displacement responses for the MBB A-3 airfoil at $M=0.765$ and $\alpha=0$ deg by USTS.

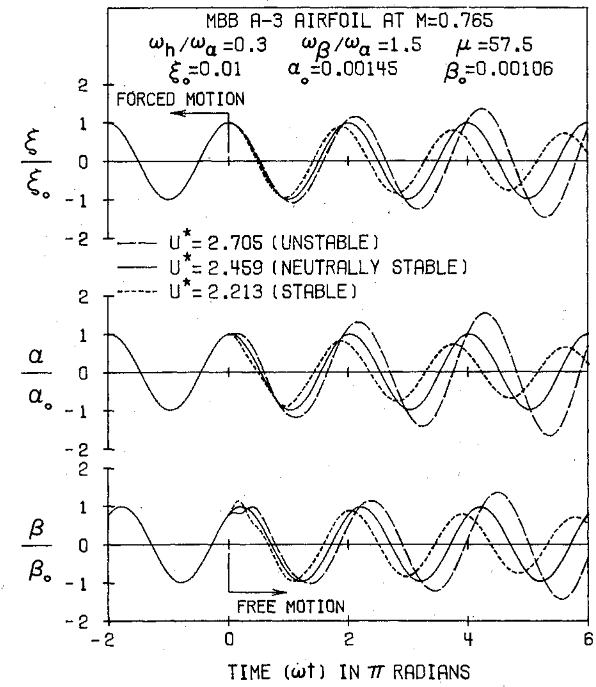

Fig. 16 Effect of flight speed on displacement responses for the MBB A-3 airfoil at $M=0.765$ and $c_l \approx 0.58$ by LTRAN2-NLR.

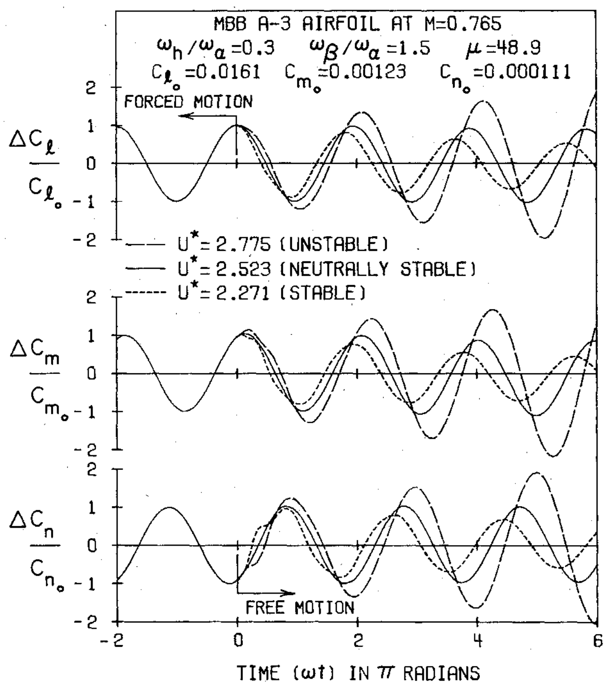

Fig. 15 Effect of flight speed on aerodynamic responses for the MBB A-3 airfoil at $M=0.765$ and $\alpha=0$ deg by USTS.

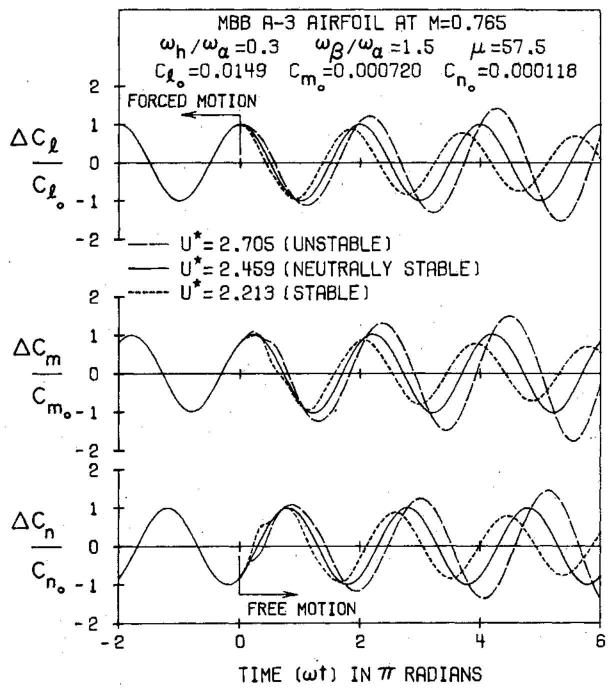

Fig. 17 Effect of flight speed on aerodynamic responses for the MBB A-3 airfoil at $M=0.765$ and $c_l \approx 0.58$ by LTRAN2-NLR.

predicted by the transonic codes. The Mach numbers differ by only 1.8%.

Numerical convergence difficulties were encountered in obtaining a steady solution from USTS. 10,000 time steps were required before the lift coefficient, oscillating in time about the steady-state value, finally converged to $c_l = 0.343$. A similar numerical oscillatory convergence problem was also experienced in the unsteady computations using USTS. Many cycles of forced motion were required to obtain fairly periodic solutions.

Flutter analyses for this case using both codes compared well. These results, along with the flight speed values for time response, are recorded in Table 2. LTRAN2-NLR response results for the three displacements and the three aerodynamic forces were plotted in Figs. 12 and 13, respectively.

Based on the flutter solution, the time responses were found to be neutrally stable. Responses for stable and unstable conditions were also determined by changing the flight speed. Small disturbances due to the initial conditions in the first cycle of free motion were again observed.

The frequency for stable responses is higher and the frequency for unstable responses is lower than that of the neutrally stable curves. In addition, the unstable responses diverge quickly due to higher relative pitching response. As pointed out by Zwaan,[3] it is the aerodynamic forces due to pitching that can drive the aeroelastic system.

USTS response histories were plotted in Figs. 14 and 15. Based on the flutter solution, the time-response curves were found to be slightly diverging. The neutrally stable responses were obtained when the flight speed was reduced 2.5%. This small difference is felt to be caused by the numerical convergence difficulties described earlier. USTS time-response results were also obtained for stable and unstable conditions by changing the flight speed, as listed in Table 2, for comparison with LTRAN2-NLR responses. Good agreement was found.

Case 2: At Design Mach Number of 0.765 and Design Lift Coefficient of 0.58

In this case, USTS was not used for time-response analyses because of numerical convergence difficulties. This case was also studied by Yang and Chen[7] using the LTRAN2-NLR code. Smaller time steps were used here, i.e., 360 steps per cycle of motion as compared to 120 steps used in Ref. 7. It was found in Ref. 7 that the value of U^* required to produce neutrally stable responses is 3.5% lower than the flutter solution. By using smaller time steps in this study, the value of U^* required to produce neutrally stable responses is exactly the same as the flutter solution. This finding indicates that the use of linear superposition of airloads in the present case gives a flutter solution which results in neutrally stable responses. These numerical results are given in Table 2. Response histories are shown in Figs. 16 and 17.

Concluding Remarks

Based on the present 3DOF time-response analyses, the following concluding remarks can be made:

1) Aeroelastic time-response analyses were performed for the NACA 64A006, NACA 64A010, and MBB A-3 airfoils in small-disturbance transonic flow. In contrast with the *U-g* flutter analysis, the time-response analysis properly accounts for nonlinear aerodynamic effects, and the principle of linear superposition of airloads is not needed.

2) Emphasis was placed upon neutrally stable responses which correspond to flutter conditions. Time-response histories were presented showing that for each case the flight speed used to obtain neutrally stable responses is either exactly or nearly the same as the flutter speed determined in the separate *U-g* flutter analysis. In addition, time responses computed at larger airfoil motion amplitudes for the case as shown in Figs. 6 and 7 seem to indicate that the small amplitude flutter speed remains valid for airfoil motion of larger amplitude.

3) In general, the steady pressure distributions computed by the two codes compared well. Agreement with experimental steady pressure data, where available, was also good. Time responses computed by LTRAN2-NLR and USTS compare well for the MBB A-3 airfoil at $M=0.765$, $\alpha=0.0$ deg. USTS time responses for the NACA 64A006 airfoil at $M=0.85$ are in good agreement with the LTRAN2-NLR results of Ref. 7. Applicability of the USTS transonic code is limited due to numerical convergence problems. USTS seems to be most applicable to thin symmetric airfoils at zero mean angle of attack. CDC 6600 computing times are approximately 1.2 and 1.4 s per time step using USTS and LTRAN2-NLR, respectively.

4) The present time-response analysis methods are being extended for the study of flutter suppression using active controls. With the inclusion of a control law relating the airfoil motion to the trailing edge control surface deflection, active flutter control of conventional and supercritical airfoils may be investigated using time-response methods.

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